Lecture Note 1: Basic Elements of Investing and Arbitrage

Investing: "An investment operation is one which, upon thorough analysis promises safety of principal and an adequate return. Operations not meeting these requirements are speculation."

Benjamin Graham – **The Intelligent Investor**, Revised Edition, p. 18.

Arbitrage is a financial arrangement whereby a security may be purchased in one market and simultaneously sold in another to generate a profit based on relative discrepancies in price between the two markets.

An example of Arbitrage:

In the Summer of 2004, Microsoft [MSFT] after settling its case with the European Common Market over anti-trust issues, announced a special $3 per share dividend to holders of shares on December 15, 2004. Microsoft had about $50 Billion in cash/near cash securities on its balance sheet and this transaction would cost the company about $30 billion leaving $20 billion still on its books. At the time of the announcement, MSFT shares were priced at $25/share. Margin interest or borrowing rates were hovering around 4%, and dividend income was going to be taxed at 15%. You had a $450,000 stock portfolio, that was marginable, and your marginal tax rate was 31%. Given these assumptions the following arbitrage opportunity presented itself:

Dividend yield after MSFT's July announcement: $3/$25 = 12%

Short-term Borrowing Costs [margin rate] = 4%

Spread without regard to taxes: 12% - 4% = 8%

==============➔ arbitrage the favorable spread up to December 15th, 2004.

Spread including taxes: 12%(1 - .15) - 4%(1 - .31) = 10.2% - 2.76% = 7.44%

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Tax Rate on Dividends — Marginal Tax Rate

7/15/2005===→12/30/2005

Borrow $90,000 @ 4%	Sell 3600 Shrs MSFT
Buy $90,000/$25 per share = 3600 MSFT Shrs.	@ $25/shrs. = $90,000
Receive: $3 x 3,600 shrs. = $10,800 in Dividends	Pay back the margin loan.

Pay: 4% X $90,000 = ($3,600) investment interest expense

Arbitrage profit = $7,200

Conclusion: This arbitrage strategy will produce a profit of $7,200 as long as MSFT stock remains at $25, however the stock can go no lower than $22/share for this arbitrage to work.

An example of Mortgage Interest Arbitrage in 2005

For 15 years you have been paying off a 6% mortgage that currently has an outstanding loan balance of $70,000. The home on which the loan is written has an assessed value of $250,000. Last year's mortgage interest deduction was $6,000 and since you are in the 31% tax bracket you would like to increase the Schedule A interest deduction while generating non-taxable income to the household. On April 15th, 2005, a 15 year fixed rate mortgage can be obtained with an 80% loan to value ratio, and $2,000 in closing costs for 5.5%. At the same time you note that the Duff and Phelps (DTF) tax exempt bond fund is generating a $.96/share return on stock priced at $14.50. Thus the tax exempt return on this security is $.96/$14.50 = 6.6%. What arbitrage opportunity exists between the 5.5% mortgage rate and the 6.6% tax exempt bond yield?

Arbitrage application

You decide to refinance $170,000 of debt on the existing home. The new loan to value ratio will be: $170,000/$250,000 = 68% which is well below the 80% threshold requirement [i.e., you have a much stronger collateral position and therefore a safer loan for the bank than what is required]. However, with the $2,000 in closing costs the total amount of financing will be: $170,000 + $2,000 or $172,000. After the loan is executed $70,000 of the older 6% loan will be paid off and a new loan of $72,000 plus $100,000 will be substituted at 5.5%. The $72,000 will replace the original $70,000 loan amount leaving $100,000 to invest in DTF.

Arbitrage Opportunity:

After Tax Cost of Debt = 5.5% (1 - .31) = 3.795%

After Tax cost of DTF Return = 6.6%

Arbitrage Spread = 6.6% - 3.795% = 2.805%

Arbitrage Profit = $100,000 x .02805 = $2,805

Interest Arbitrage Using Near Term Futures on a 5-Year Treasury Bond

Exploiting an inverted-flat yield curve can present an arbitrage opportunity using treasury futures.

Consider the Daily Treasury Yield Curve Rates as published at the US Treasury Website on 1/6/2006

Date	1 Month	3 Month	6 Month	5 Year

1/6/2006	4.06	4.22	4.39	4.32

In this case the Short-term 6-month Yield [4.39%] > 5-Year Treas. Yield [4.32%]

Potential Arbitrage Action: Short the spot market on a $100,000 5-year T-Bond and then go long the near term 5-Year Futures contract, Unwind the position 3 months later to earn a profit on the relative pricing difference.

Note: This cash and carry arbitrage depends on the relative difference in pricing caused by an unusual inverted yield curve situation where short term rates are above long term rates for interest bearing treasury securities. It is important that the security in the cash (spot) market match up with the underlying security used in the futures contract.

5-Year US Treasury Note Futures on 1/6/2006

Expiration Date	Open Bid	Closing Bid	FROM THE CBOT
March 15, 2006	106'165	106'170	

US Treasury Bond Note Prices/Yields from Yahoo Bond Screener

US Treasury Note 3.5% due Jan 15, 2011

Price	Coupon	Yield
107.39	3.5	1.945

Cash and Carry Arbitrage Transaction:

Assume that since you are completing this transaction on 1/6/2006 there is only 6 days of coupon interest so that accrued interest is not a significant factor in this transaction. Buying the futures contract at the beginning of a coupon interest period means the price going into the futures position will reflect coupon interest.

1/6/2006	3/15/2006
Open	Close Out Position
Short [Sell] ===➔	Buy [Long]
$100,000 US Treas. 3.5% Coupon	$100,000 US Treas. Bond

Bond due 2011 and take proceeds to ====================➔ Sell [short] the Futures Contract

Buy 1 [go long] Futures Contract 5-year T-Note Expiring 3/15/2006 to Buy the $100,000 Treas. Bond

Steps to the Arbitrage:

On 1/6/2006 Short the US Treas. Bond @ $107.39/$100 of face amount

⇨ $107.39 X 1000 = $107,390 in Sale Proceeds.

Take the cash to Buy 1 Futures Contract @106.17 ➔ $106 + 17/32 = $106.3125

3 Months Later

On 3/15/06 you unwind the arbitrage by selling the futures contract taking the proceeds to cover your short bond position.

The difference between the initial spot sale less the cost of the futures contract:

$107,390

($106,531.25)

$ 858.75 represents the advantage in these positions

Setting Limits on Option Premiums

Arbitrage occurs whenever two similar assets sell for widely different prices in two markets.

Because financial assets tend to be uniform and homogeneous in structure – arbitrage opportunities become readily apparent – which is why they don't last long where markets tend to be efficient.

It should be noted though that not all markets are efficient – largely due to externalities that may last a long time. Nevertheless in the case of futures markets, pricing is quite efficient and therefore arbitrage opportunities are short lived.

Financial markets allow one to combine different securities together to produce identical payment patterns as another security --- the combined securities are called **synthetic securities**.

These other alternative securities force strict pricing limits among securities to prevent long-term arbitrage. When comparing differences among payment patterns it is important to be able to determine future [or forward]rates based on the term structure of interest. Forward rates [or yields] are implied by current cash yields based on term structure of interest rates.

$$[1 + r_{o,L}]^L = [1 + r_{0,s}]^s [1 + r_{s,L}]^{L-s}$$

$r_{o,L}$ = cash yield on long-term security with maturity L

$r_{0,s}$ = cash $_{\text{yield}}$ on a short-term security with maturity s

$r_{s,L}$ =implied forward rate between s and L

E.G. Assume a cash yield on a 1-year T-Bill is 4.5%. A two year Treasury Note yields an average annual return of 4.7% [positive yield curve]. What must be earned in the T- Bill market in year 2 to be an equivalent investment to the Treasury Note covering the entire 2 year period?

$$(1 + .047)^2 = (1 + .045)^1 (1 + r_{s,2})^{2-1}$$

$$[(1.047)^2/(1.045)] - 1 = r_{s,2}$$

$$r_{s,2} = 4.9\%$$

Problem: What is the implied forward rate from 6 to 9 months, if a 9 month T-Bill yields 6.5% and a six month T-Bill yields 6.25%?

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0 1 2 3

0	¼	½	¾
	3 months	6 months	9 months

Basic Relationship:

$(1 + .065/4)^{3/4} = (1 + .0625/4)^{1/2} (1 + r/4)^{1/4}$

$(1.01625)^{.75} = (1.015625)^{.5} (1 + r/4)^{1/4}$

$1.012162910 = 1.007782219 (1 + r/4)^{1/4}$

$[1.01216910]/[1.007782219] = (1 + r/4)^{1/4}$

$1.004346863 = (1 + r/4)^{1/4}$

$[1.004346863]^4 = 1 + r/4$

$4[.017501153] = r$

$.070004612 = r$

$r = 7\%$

A difference between the implied forward rate and actual rates on intermediate securities would imply that an arbitrage opportunity may exist.

Theorem 1 [lower bound, min values for call premiums]

At expiration, the call premium can be no smaller than either 0 or the difference between the underlying stock price and the fixed exercise price on the option.

$C_t = \text{Max} \{ 0, S_t – X\}$

Where,

C_t = Call premium that exists on the expiration date t

S_t = Stock price on the date of expiration t

X = the fixed exercise price that remains constant over the life of the option contract all the way to expiration.

A zero call premium ➔ represents an out of the money call, with the stock price at expiration that is below the exercise price. If you exercised the option, paid the exercise price and bought the stock you would be paying more than the stock was currently worth.

E.G. Consider the following call option information on Home Depot [HD]

The HD stock price is =➔ $43

The exercise (strike) price on the call option is ==➔ $40

The call premium ====➔ $4.50

What type of trading strategy could be used to produce an arbitrage profit?

Note: Strike Price - Exercise price < Call premium

$43 - $40 < $4.50

This call option is out of the money – if you bought the call at $4.50 and then exercised at $40 you be paying $4.50 + $40 = $44.50 for the stock which is greater than $43 the current price of HD shares.

Since the call is out of the money – you want to consider writing a call.

===➔ Buy HD stock at $43 and write a covered call and pick up the call premium of $4.50.

If the stock rises above $43 + $4.50 = $47.50 it will get called away. In this case you have effectively bought a security for $43 and then sold it for $47.50.

If the stock stays at \$47.50 or less the call option will be out of the money and expire without being exercised and you pick up the \$4.50 and retain the stock.

Theorem 2 [upper bound for the Call premium]

The call premium can never be more than the value of the underlying stock.

Polar Case: Lowest possible strike price is 0.

If $X=0$, $C= S-X \Longrightarrow C=S$

Therefore, the highest value for C would be S.

$C_t = S_t - X$

Theorem 3 [You pay a premium for more time in an option]

Given two options that are identical in terms of having the same exercise price and underlying asset, the option with the longer time to expiration will have a higher .

If $t_1 > t_2$, the $C_1 > C_2$.

Theorem 4 [There is an inverse relationship between the exercise [strike] price and the call premium]

Given two options that are the same in all respects except one has a higher exercise price, then it must have a lower call premium.

When $X_1 > X_2$ then $C_1 < C_2$. If $C_t = S_t - X$, then let $S_t = S$, the current stock price. Whenever X increases, $C_t = S_t - X$ [increasing] $\Rightarrow C_t$ will decline.

Theorem 5 [lower bound on the Call Premium on a present value basis]

Prior to expiration, the call premium [C_t] will not be less than either 0 or the current stock price minus the present value of the exercise [strike] price.

C_t => Max { 0, $S_t - X e^{-rt}$ }

Case 1: If the call is out of the money, the call premium C_t = 0, and the call is worthless.

Case 2: If the call is not out of the money, then the current value of the stock, S_t must be equal to the present value of the cash flows derived from the call option representing the stock.

Stock Price [S_t] = call premium [C_t] + X[payment required to acquire the stock] e^{-rt}[discount factor]

Which then can be arranged to produce:

[C_t] = [S_t] - $X e^{-rt}$.

Theorem 6 [The call option provides unlimited upside return while limiting downside loss]

The higher(lower) the risk of the underlying asset on which the option is being written, the higher(lower) the call premium.

In other words, if the st. dev. of the share price of stock A > st. dev. of the share price of stock B, then

the call premium of stock A will be > than the call premium for stock B, indicating the higher return potential on the option for A over B. Greater the variance in the stock price, the better than chance for a gain from holding the call option.

Theorem 7 [In the absence of a dividend, time matters]

When a stock does not pay a dividend during the life of the option, then early exercise is never optimal.

Theorem 8 [Dividend paying stocks may present an opportunity to exercise early on the call option]

When the stock pays a dividend during the life of the option, the early exercise may be optimal if the dividend plus interest exceeds the interest that could be earned on the exercise price.